Acoustic decoherence of flux qubits

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Decoherence of a flux qubit due to inelastic scattering of thermal phonons by the qubit is studied. The computed decoherence rates contain no unknown constants and are expressed entirely in terms of measurable parameters of the qubit. The answer depends strongly on the size of the qubit as compared to the wavelength of phonons of frequency $f = \Delta/(2\pi\hbar)$, with Δ being the tunnel splitting of the qubit. Thermal phonons set the upper limit on the operating temperature of a small qubit at around $10-20$K. For large qubits acoustic decoherence due to one phonon processes should be taken into account at any temperature when a high quality factor is desired.

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I. INTRODUCTION.

Flux qubits have received much attention lately due to low decoherence exhibited at a macrosopic scale. In essence such a qubit consists of a small superconducting loop interrupted by one or more Josephson junctions. When half a flux quantum is applied to the loop, the system forms a symmetric double-well potential. The two classical states associated with the minima of this potential correspond to clockwise and counterclockwise circulating currents. The fascinating property of such a system is that it exhibits quantum tunneling between the classical minima [1, 2, 3, 4], thus, providing an example of a quantum superposition of macroscopic current states. The ground state and the first excited state of the system are symmetric and antisymmetric superpositions of clockwise and counterclockwise current states, separated by the tunnel splitting Δ. When the system is prepared, e.g., in a clockwise current state, the probability to find it with clockwise or counterclockwise current oscillates coherently at a frequency $f = \Delta/(2\pi\hbar)$. Experiments have demonstrated that the quality factor of the flux qubit can be as high as 5000 [5], thus, pushing it to the front line of promising candidates for quantum computation. Recently, this promise has been further amplified by successful experiments with coupled flux qubits [6, 7, 8].

Most of the work on decoherence of flux qubits concentrates on the effects of non-zero impedance [12], electromagnetic [1, 9, 10, 11] and $1/f$ noise [12, 13]. Decoherence due to coupling of a flux qubit to the acoustics waves has been studied to a lesser degree [9]. Meantime, such a decoherence is generic for a flux qubit. This can be seen from the following argument. Tunnelling between clockwise and counterclockwise circulation is accompanied by the reversal of the angular momentum associated with the circular current. In experiments on flux qubits, this angular momentum can be as large as $10^{10}\hbar$, so that its non-conservation would be quite dramatic. A similar problem exists for any LC circut. To conserve the angular momentum, a freely suspended ring containing an inductor and a capacitor would have to co-wiggle mechanically with the oscillating current. When such a ring is attached to a solid matrix it should produce torsional oscillations in the matrix. This immediately suggests that unless the flux qubit is in the ground state, it should have a finite probability to radiate a phonon. The latter can be viewed as a consequence of the fact that the electric current is defined with respect to the crystal lattice of ions so that quantum states of the current are inevitably entangled with the lattice states.

It was shown earlier [9] that the effective coupling of the oscillating current with the crystal lattice has an universal form that does not contain any fitting parameters. At very low temperatures the typical decoherence rate due to this coupling ranges from 1s^{-1} for a small qubit to 10^6s^{-1} for a large qubit. The low temperature decoherence rate is due to a spontaneous decay of an excited current state to the lower energy state, accompanied by the emission of a phonon. Such processes are known as one phonon processes. They dominate relaxation and decoherence due to phonons in the low temperature limit. Since operating qubits at elevated temperatures is desirable one must consider other phonon processes which may be dominant at these temperatures. Such processes consist of inelastic scattering of thermal phonons by the qubit. They are known as two phonon processes. In this paper, we focus on the decoherence via a two phonon channel. We find that the significance of this channel strongly depends on the size of the qubit, R, as compared to the wavelength, $\lambda = v_t/f$, of a phonon of frequency $f = \Delta/(2\pi\hbar)$ (with v_t being the speed of the transverse sound). It turns out that for a small qubit the two phonon processes can be the main factor limiting the qubit operating temperature. Another side of the coin, however, is that the corresponding two phonon rate is proportional to the square of the energy bias between the two states with opposite directions of the circulating current. This, in principle, allows one to switch the two phonon decoherence on and off by changing the external magnetic flux that controls the bias. For a large qubit the one phonon processes seem to always win over the two-phonon processes.

The structure of the paper is as follows. Interaction between superconducting current and elastic twists is discussed in Section II. Two phonon processes are studied in

Section III. Subsection III-A deals with inelastic scattering of thermal phonons by small qubits. Matrix elements and decoherence rates for large qubits are studied in Section III-B. Section IV contains comparison of one-phonon and two-phonon rates, as well as numerical estimates of the acoustic decoherence rate, and general discussion of the results.

II. COUPLING OF SQUID STATES TO ACOUSTIC PHONONS

We consider a loop of radius R carrying a current J oscillating with frequency $f_0 \sim 10^9 - 10^{10}\mathrm{s}^{-1}$ between the clockwise and counterclockwise direction of motion. The angular momentum associated with the circular current changes from L to $-L$ every half a period. As a consequence, the local environment co-wiggles with the same frequency and produces elastic distortions of the lattice in accordance with angular momentum conservation. These distortions, $\mathbf{u}(\mathbf{r}, t)$, being pure twists, satisfy

$$\nabla \cdot \mathbf{u} = 0. \tag{1}$$

The kinetic energy associated with the superconducting current can be written as

$$KE = \frac{n_e m_e \mathbf{v}_e^2}{2} = \frac{n_e m_e}{2}(\mathbf{v}_{Lat} + \dot{\mathbf{u}})^2, \tag{2}$$

where $\mathbf{v}_e$ is the velocity field of electrons in the laboratory coordinate frame, $\mathbf{v}_{Lat}$ is same velocity in the lattice frame, n_e is the concentration of electrons, and m_e is electron mass. Interaction of the current with lattice distortions can be formulated by noting that the current is defined in the frame of the moving ions,

$$\mathbf{j} = en_e\mathbf{v}_{Lat} = en_e(\mathbf{v}_e - \dot{\mathbf{u}}), \tag{3}$$

where e is electron charge. From Eq. (2) one obtains [9]:

$$\mathcal{H}_{int} = \frac{m_e}{e}\int d^3r \mathbf{j}\cdot\dot{\mathbf{u}}. \tag{4}$$

This Hamiltonian satisfies all the symmetries and is free of any unknown interaction constants.

We denote the eigenstates of the angular momentum, $L_z = L$ and $L_z = -L$, as $|\uparrow\rangle$ and $|\downarrow\rangle$, respectively. The system under consideration can be approximately modeled as a particle of spin L in a biased double well potential described by the Hamiltonian $\mathcal{H}_0$ whose lowest eigenstates are

$$|0\rangle = \frac{1}{\sqrt{2}}(C_-|\uparrow\rangle + C_+|\downarrow\rangle) \tag{5}$$

and

$$|1\rangle = \frac{1}{\sqrt{2}}(C_+|\uparrow\rangle - C_-|\downarrow\rangle). \tag{6}$$

Here

$$C_\pm = \sqrt{1 \pm \varepsilon/\Delta}, \tag{7}$$

where

$$\Delta = \sqrt{\Delta_0^2 + \varepsilon^2} \tag{8}$$

is the energy splitting of $|0\rangle$ and $|1\rangle$, ε is the bias, and Δ_0 is the energy splitting at $\varepsilon = 0$.

The total Hamiltonian of the system is

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{ph} + \mathcal{H}_{int}. \tag{9}$$

The first two terms stand for the interaction-free qubit and phonon Hamiltonian, respectively, and the last term is given by Eq. (4). Decoherence of a general superposition state $|\Psi\rangle = c_1|0\rangle + c_2|1\rangle$ will take place through relaxation of $|1\rangle$ to $|0\rangle$. The corresponding two phonon transition is determined by the matrix element of $\mathcal{H}_{int}$ that will be computed in the next Section. To shorten formulas, unless stated otherwise, we will use the units in which $k_B = \hbar = 1$.

III. TWO-PHONON PROCESSES

A. Small qubit.

In the case of a small flux qubit of size $R \ll \lambda$, one can treat the local environment at the position of the qubit as a rigid matrix making a tiny local rotation with angular velocity [14]

$$\mathbf{\Omega} = \frac{1}{2}\nabla\times\dot{\mathbf{u}}. \tag{10}$$

Equivalently, the velocity vector $\dot{\mathbf{u}}$ can be expressed in terms of the angular velocity as $\dot{\mathbf{u}} = \mathbf{\Omega}\times\mathbf{r}$, where $\mathbf{r}$ is the position vector with its origin at the center of the qubit. The interaction Hamiltonian of Eq. (4) can now be simplified as

$$\mathcal{H}_{int} = \mathbf{L}\cdot\mathbf{\Omega}, \tag{11}$$

in which

$$\mathbf{L} = \frac{m_e}{e}\int d^3r\ \mathbf{r}\times\mathbf{j} \tag{12}$$

is the angular momentum of the circulating current. Hamiltonian of Eq. (11) is a consequence of the conservation of angular momentum.

We are interested in the process of relaxation through a two phonon channel in which one phonon is absorbed and another is emitted with wavevectors $\mathbf{k}$ and $\mathbf{q}$ respectively. The matrix element for this process is

$$\begin{aligned} M \ = \ & \sum_\xi \frac{\langle\Psi_f|\,\mathcal{H}_{int}\,|\psi_\xi\rangle\,\langle\psi_\xi|\,\mathcal{H}_{int}\,|\Psi_i\rangle}{E_1\ +\omega_{\mathbf{k}}\ - E_\xi} \\ + & \sum_\xi \frac{\langle\Psi_f|\,\mathcal{H}_{int}\,|\psi_\xi\rangle\,\langle\psi_\xi|\,\mathcal{H}_{int}\,|\Psi_i\rangle}{E_1\ -\omega_{\mathbf{q}}\ - E_\xi}, \end{aligned} \tag{13}$$

where $|\Psi_i\rangle$ ($|\Psi_f\rangle$) stand for the initial (final) state of the system, $\omega_{\mathbf{k}}$ and $\omega_{\mathbf{q}}$ are phonon frequencies, and E_1 is

the energy of the first excited state $|1\rangle$. The summation over ξ labels the energy states of $\mathcal{H}_0$, and the intermediate phonon states $|n_{\mathbf{k}}-1, n_{\mathbf{q}}\rangle$ in the first term and $|n_{\mathbf{k}}, n_{\mathbf{q}}+1\rangle$ in the second term. With the help of Eq. (10) and conventional quantization of the phonon field,

$$\mathbf{u} = \sqrt{\frac{1}{2\rho V}} \sum_{\mathbf{k}\lambda} \frac{\mathbf{e}_{\mathbf{k}\lambda} e^{i\mathbf{k}\cdot\mathbf{r}}}{\sqrt{\omega_{\mathbf{k}\lambda}}} \left(a_{\mathbf{k}\lambda} + a^{\dagger}_{-\mathbf{k},\lambda} \right), \tag{14}$$

where ρ is the mass density of the solid matrix and $\mathbf{e}_{\mathbf{k}\lambda}$ is the polarization vector of the phonon, one obtains for the phonon part of the matrix element

$$\begin{aligned} M_{ph} &= \langle n_{\mathbf{q}}+1| \Omega_z |n_{\mathbf{q}}\rangle \langle n_{\mathbf{k}}-1| \Omega_z |n_{\mathbf{k}}\rangle \\ &= \langle n_{\mathbf{k}}-1| \Omega_z |n_{\mathbf{k}}\rangle \langle n_{\mathbf{q}}+1| \Omega_z |n_{\mathbf{q}}\rangle \\ &= \frac{1}{8\rho V} [\mathbf{k}\times\mathbf{e}_{\lambda}]_z [\mathbf{q}\times\mathbf{e}_{\sigma}]_z \sqrt{\omega_{\mathbf{k}\lambda}\omega_{\mathbf{q}\sigma}(n_{\mathbf{q}}+1)n_{\mathbf{k}}}. \end{aligned} \tag{15}$$

The action of the angular momentum operator on $|0\rangle$ and $|1\rangle$ produces the following states:

$$\begin{aligned} L_z|0\rangle &= \frac{L\Delta_0}{\Delta}\left[|1\rangle - \frac{\varepsilon}{\Delta_0}|0\rangle\right] \\ L_z|1\rangle &= \frac{L\Delta_0}{\Delta}\left[|0\rangle + \frac{\varepsilon}{\Delta_0}|1\rangle\right]. \end{aligned} \tag{16}$$

Inserting these results into Eq. (13) one can immediately see that $|\psi_{\xi}\rangle$ must be either $|0\rangle$ or $|1\rangle$. Thus, we only need to consider

$$\begin{aligned} -\langle 0| L_z |0\rangle &= \langle 1| L_z |1\rangle = \frac{L\varepsilon}{\Delta} \\ \langle 0| L_z |1\rangle &= \langle 1| L_z |0\rangle = \frac{L\Delta_0}{\Delta}. \end{aligned} \tag{17}$$

Taking into account the conservation of energy, $\omega_{\mathbf{q}} - \omega_{\mathbf{k}} = \Delta$, the matrix element (13) reduces to

$$M = M_{ph}\frac{2L^2\varepsilon\Delta_0}{\Delta^2}\frac{1}{\omega_{\mathbf{k}}(\omega_{\mathbf{k}}+\Delta)}. \tag{18}$$

The relaxation rate can be obtained using the Fermi golden rule in the second order, given by

$$\Gamma_2 = \sum_{\lambda,\sigma}\int \frac{d\mathbf{k}d\mathbf{q}}{(2\pi)^6} V^2 |M|^2 2\pi\delta(\omega_{\mathbf{q}} - \omega_{\mathbf{k}} - \Delta). \tag{19}$$

Substitution of Eq. (18) into Eq. (19) yields

$$\begin{aligned} \Gamma_2 &= \frac{L^4\varepsilon^2\Delta_0^2}{16\rho^2\Delta^2(2\pi)^5}\sum_{\lambda,\sigma}\int d\mathbf{k}d\mathbf{q}[(\mathbf{k}\times\mathbf{e}_{\lambda})_z(\mathbf{q}\times\mathbf{e}_{\sigma})_z]^2 \\ &\times \frac{\omega_{\mathbf{q}}(n_{\mathbf{q}}+1)n_{\mathbf{k}}}{\omega_{\mathbf{k}}(\omega_{\mathbf{k}}+\Delta)^2}\delta(\omega_{\mathbf{q}} - \omega_{\mathbf{k}} - \Delta). \end{aligned} \tag{20}$$

Replacing the integration variable $d\mathbf{k}$ with $\omega_{\mathbf{k}}^2 d\omega_{\mathbf{k}} d\Omega_{\hat{\mathbf{k}}}/v_i^2$ and the vector $\mathbf{k}$ with $\hat{\mathbf{k}}\omega_{\mathbf{k}}/v_i$ and $\hat{\mathbf{k}} = \mathbf{k}/k$, one obtains

$$\begin{aligned} \Gamma_2 &= \frac{L^4\varepsilon^2\Delta_0^2 B^2}{16\rho^2\Delta^2(2\pi)^5} \\ &\times \int_0^{\omega_D} \frac{d\omega\omega^3(\omega+\Delta)^3}{(e^{\omega}-1)(1-e^{-(\omega+\Delta)/T})}, \end{aligned} \tag{21}$$

where

$$B = \sum_{\lambda}\int \frac{d\Omega_{\hat{\mathbf{k}}}}{v_{\lambda}^5}(\hat{\mathbf{k}}\times\mathbf{e}_{\lambda})_z^2. \tag{22}$$

The sum in Eq. (22) runs over the two transverse polarizations. Making the change of variable $x = \omega/T$ yields

$$\Gamma_2 = \frac{L^4\varepsilon^2\Delta_0^2 B^2 T^7}{16(2\pi)^5\rho^2\Delta^2} g(\Delta/T), \tag{23}$$

where

$$g(\Delta/T) = \int_0^{\infty} \frac{dx x^3(x+\Delta/T)^3}{(e^x-1)(1-e^{-x-\Delta/T})}. \tag{24}$$

For completeness, we insert k_B and $\hbar$ into our result which in the limit $k_B T \ll \Delta$ reduces to

$$\Gamma_2 \cong \frac{\pi}{270}\frac{L^4\varepsilon^2\Delta_0^2\Delta k_B^4 T^4}{\rho^2 v_t^{10}\hbar^7}. \tag{25}$$

In the opposite limit $k_B T \gg \Delta$ one obtains for the rate

$$\Gamma_2 \cong \frac{15\pi}{36}\frac{L^4\varepsilon^2\Delta_0^2 k_B^7 T^7}{\rho^2\Delta^2 v_t^{10}\hbar^7}. \tag{26}$$

B. Large qubit.

For a large qubit, $R \gg \lambda$, one must integrate the interaction of the superconducting current with phonons along the entire loop as in Eq. (4),

$$\mathcal{H}_{int} = \frac{im_e}{e}\sum_{\mathbf{k}\lambda}\sqrt{\frac{\omega_{\mathbf{k}}}{2V\rho}}(\mathbf{j}_{\mathbf{k}}\cdot\mathbf{e}_i)(a_{\mathbf{k}\lambda} - a\dagger_{\mathbf{k}\lambda}), \tag{27}$$

where $\mathbf{j}_{\mathbf{k}} = \int dr^3 \mathbf{j} e^{i\mathbf{k}\cdot\mathbf{r}}$ is the Fourier transform of $\mathbf{j}$. The solution to $\mathbf{j}_{\mathbf{k}}$ in the thin-ring approximation is

$$\mathbf{j}_{\mathbf{k}} = -i2\pi R J_1(k_{\perp}, R) J \mathbf{n}_{\mathbf{k}}, \tag{28}$$

where $\mathbf{n}_{\mathbf{k}} \perp \mathbf{k}$ is a unit vector in the plane of the ring, $k_{\perp}$ is the z-component of $\mathbf{k}$, and $J_1(k_{\perp}, R)$ stands for the Bessel function of the first order. The interaction Hamiltonian can now be written as

$$\mathcal{H}_{int} = \mathbf{L}\cdot\mathbf{\Omega}_{eff}, \tag{29}$$

where

$$\mathbf{\Omega}_{eff} = -i\pi\frac{R}{a}\sum_{\mathbf{k}}\sqrt{\frac{\omega_{\mathbf{k}}}{2V\rho}} J_1(k_{\perp}, R)(a_{\mathbf{k}t} - a\dagger_{\mathbf{k}t})\mathbf{e}_z. \tag{30}$$

The scalar product $\mathbf{n}_{\mathbf{k}}\cdot\mathbf{e}_{\lambda} = \delta_{t\lambda}$ as $\mathbf{n}_{\mathbf{k}}$ is in the plane of the ring. To obtain the two phonon matrix element one needs only to replace $(\mathbf{k}\times\mathbf{e}_{\lambda})_z(\mathbf{q}\times\mathbf{e}_{\sigma})_z$ in Eq. (15) with

$$-\left(\frac{2\pi R}{a}\right)^2 J_1(k_{\perp}R) J_1(q_{\perp}R). \tag{31}$$

Then, the rate for a large qubit becomes

$$\Gamma_2 = \frac{\pi L^4 \varepsilon^2 \Delta_0^2 R^4}{8\rho^2 \Delta^2 a^4 v_t^6} \int_0^{\omega_D} d\omega f(\omega, \Delta, T)\Theta(\omega, R, v_t, \Delta), \tag{32}$$

where

$$f(\omega, \Delta, T) = \frac{\omega(\omega+\Delta)}{(e^{\omega/T}-1)(1-e^{-(\omega+\Delta)/T})}, \tag{33}$$

and

$$\begin{aligned}\Theta(\omega, R, v_t, \Delta) &= \int_0^\pi d\theta_1 \sin\theta_1 J_1^2\left(\frac{\omega R}{v_t}\sin\theta_1\right) \\ &\times \int_0^\pi d\theta_2 \sin\theta_2 J_1^2\left(\frac{[\omega+\Delta]R}{v_t}\sin\theta_2\right).\end{aligned} \tag{34}$$

Due to its large argument, $J_1([\omega+\Delta]R/v_t \sin\theta_2)$ can be approximated by its asymptotic form. The integral over θ_2 in Eq. (34) then yields approximately $v_t/(\omega+\Delta)R$. The integral over θ_1 equals

$$\frac{1}{3}\left(\frac{\omega R}{v_t}\right)^2 {}_1F_2\left(\frac{3}{2}, \frac{5}{2}, 3, -\left(\frac{\omega R}{v_t}\right)^2\right), \tag{35}$$

where ${}_1F_2$ is the generalized hypergeometric function. Inserting the results just obtained into Eq. (32) and making the change of variable $x = \omega/T$, one obtains

$$\Gamma_2 \cong \frac{\pi L^4 \varepsilon^2 \Delta_0^2 R^5 T^4}{24\rho^2 \Delta^2 a^4 v_t^7} \int_0^{\omega_D/T} dx \frac{x^3(e^x-1)^{-1}{}_1F_2(-\beta^2 x^2)}{(1-e^{-x-\Delta/T})}, \tag{36}$$

where $\beta = RT/v_t$. In the limit $T \gg \Delta$, the integral in Eq. (36) simplifies to

$$\int_0^\infty dx \frac{x^3 e^x {}_1F_2(-\beta^2 x^2)}{(e^x-1)^2}, \tag{37}$$

or, after integrating by parts

$$\int_0^\infty dx \frac{x^3 e^x {}_1F_2(-\beta^2 x^2)}{(e^x-1)^2} = \frac{6}{\beta^2}\int_0^\infty dx \frac{J_2(2\beta x)}{e^x-1}. \tag{38}$$

For $\beta \gg 1$, Eq. (38) reduces to $3\beta^{-2}$. The rate then becomes

$$\Gamma_2 \cong \frac{\pi L^4 \varepsilon^2 \Delta_0^2 R^3 k_B^2 T^2}{8\rho^2 \Delta^2 a^4 v_t^5 \hbar^2}, \tag{39}$$

where we again inserted k_B and $\hbar$ into our result. In the limit $k_B T \ll \Delta$, one obtains

$$\Gamma_2 \cong \frac{\pi L^4 \varepsilon^2 \Delta_0^2 R^5 k_B^4 T^4}{24\rho^2 \Delta^2 a^4 v_t^7 \hbar^4} K\left(\frac{R k_B T}{\hbar v_t}\right), \tag{40}$$

where

$$K(y) = \int_0^\infty \frac{x^3 {}_1F_2(-y^2 x^2)}{e^x - 1} dx. \tag{41}$$

IV. DISCUSSION

We are interested in the transition temperature between the rate due to a direct process and the Raman process. The former was calculated in Ref. 9. It was shown that

$$\Gamma_1 = \frac{L^2 \Delta^5}{12\pi\rho v_t^5 \hbar^4}\coth\left(\frac{\Delta}{2k_B T}\right) \tag{42}$$

for a small qubit and

$$\Gamma_1 = \frac{L^2 \Delta^2}{4\pi R^3 \rho v_t^2 \hbar}\coth\left(\frac{\Delta}{2k_B T}\right) \tag{43}$$

for a large qubit.

In the limit $k_B T \ll \Delta$, equations (25) and (42) yield for a small qubit

$$\frac{\Gamma_2}{\Gamma_1} = \frac{12\pi^2 L^2 \varepsilon^2 \Delta_0^2 k_B^4 T^4}{270\rho \Delta^4 v_t^5 \hbar^3}. \tag{44}$$

For, e.g., $L \sim 10^2$, $\rho \sim$5g/cm^3, $v_t \sim 5\times10^3$m/s, and $f_0 \sim 5\times10^9$s^{-1}, the ratio in Eq. (44) yields $10^{-8}T^4$. Clearly, for $k_B T \ll \Delta$ ($T \sim 10^{-2}$K), the two phonon process is utterly insignificant. In the opposite limit $k_B T \gg \Delta$, equations (26) and (42) give

$$\frac{\Gamma_2}{\Gamma_1} = \frac{90\pi^2 L^2 \varepsilon^2 \Delta_0^2 k_B^6 T^6}{\rho \Delta^6 v_t^5 \hbar^3}. \tag{45}$$

For the same parameters as considered above, Eq. (45) yields $10^{-2}T^6$. The two phonon process thus dominates over a one phonon process above $T \sim 2$K. At $T \sim 20$K the decoherence rate due to two phonon processes is of order 10^6s^{-1} and, therefore, its contribution to the quality factor of the qubit cannot be ignored.

For a large qubit in the small temperature limit one obtains with the help of equations (40) and (43)

$$\frac{\Gamma_2}{\Gamma_1} = \frac{L^2 \varepsilon^2 \Delta_0^2 k_B^4 T^4}{6\pi^2 \rho \Delta^4 v_t^5 \hbar^3} K\left(\frac{R k_B T}{\hbar v_t}\right). \tag{46}$$

For a large qubit parameters: $L = 10^{10}$, $R \sim 10^{-4}$m and $T \sim 10^{-2}$K, Eq. (46) yields 10^{-5}. As in the case of a small qubit, the two phonon rate is negligible compared to the one phonon rate. In the large temperature limit equations (39) and (43) give

$$\frac{\Gamma_2}{\Gamma_1} = \frac{L^2 \varepsilon^2 \Delta_0^2 k_B T}{4\pi^2 R^2 \rho \Delta^3 v_t^3 \hbar}, \tag{47}$$

which yields $10^{-6}T$. Evidently, for a large qubit, one phonon processes are the dominant source of decoherence at any temperature. For the numbers used above the one phonon rate is of order 10^6s^{-1}.

A few important observations for flux qubits follow from the above results. Firstly, acoustic decoherence should definitely be taken into account when the quality factor as large as 10^4 is desired. Secondly, for small

biased qubits the inelastic scattering of thermal phonons by the qubit can become a dominant mechanism of decoherence above 20K. Finally, the proportionality of this mechanism to the bias provides an additional way to control the flux qubit. The interesting feature of the above results is that they do not contain any unknown constants – the decoherence rate is expressed entirely in terms of measurable parameters of the qubit.

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